6. $\int_0^{2\pi}\int_0^1\int_{-1/2}^{1/2}(r^2\sin^2\theta+z^2)\,dz\,r\,dr\,d\theta=\int_0^{2\pi}\int_0^1\left(r^3\sin^2\theta+\frac{r}{12}\right)dr\,d\theta=\int_0^{2\pi}\left(\frac{\sin^2\theta}{4}+\frac{1}{24}\right)d\theta=\frac{\pi}{3}$

12. (a) $\int_0^{2\pi}\int_0^1\int_r^{2-r^2}dz\,r\,dr\,d\theta$

(b) $\int_0^{2\pi}\int_0^2\int_0^{f(z)}r\,dr\,dz\,d\theta$ where $f(z)=\begin{cases}z, & \text{if } 0\le z\le 1\\ \sqrt{2-z}, & \text{if } 1\le z\le 2\end{cases}$

(c) $\int_0^1\int_r^{2-r^2}\int_0^{2\pi}r\,d\theta\,dz\,dr$

14. $\int_{-\pi/2}^{\pi/2}\int_0^1\int_0^{r\cos\theta}r^3\,dz\,dr\,d\theta=\int_{-\pi/2}^{\pi/2}\int_0^1 r^4\cos\theta\,dr\,d\theta=\frac{1}{5}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta=\frac{2}{5}$

18. $\int_{-\pi/2}^{\pi/2}\int_{\cos\theta}^{2\cos\theta}\int_0^{3-r\sin\theta}f(r,\theta,z)\,dz\,r\,dr\,d\theta$

20. $\int_{\pi/4}^{\pi/2}\int_0^{\cos\theta}\int_0^{2-r\sin\theta}f(r,\theta,z)\,dz\,r\,dr\,d\theta$

26. $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sec\phi}\rho^3\sin\phi\cos\phi\,d\rho\,d\phi\,d\theta=\frac{1}{4}\int_0^{2\pi}\int_0^{\pi/4}\tan\phi\sec^2\phi\,d\phi\,d\theta$

$=\frac{1}{4}\int_0^{2\pi}\left[\frac{1}{2}\tan^2\phi\right]\Big|_0^{\pi/4}d\theta=\frac{1}{8}\int_0^{2\pi}d\theta=\frac{\pi}{4}$

32. (a) $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sec\phi}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

(b) $\int_0^{2\pi}\int_0^1\int_0^{\pi/4}\rho^2\sin\phi\,d\phi\,d\rho\,d\theta+\int_0^{2\pi}\int_1^{\sqrt{2}}\int_{\cos^{-1}(1/\rho)}^{\pi/4}\rho^2\sin\phi\,d\phi\,d\rho\,d\theta$

38. $V=\int_0^{2\pi}\int_{\pi/3}^{\pi/2}\int_0^2\rho^2\sin\phi\,d\rho\,d\phi\,d\theta=\frac{8}{3}\int_0^{2\pi}\int_{\pi/3}^{\pi/2}\sin\phi\,d\phi\,d\theta=\frac{8}{3}\int_0^{2\pi}\left[-\cos\phi\right]_{\pi/3}^{\pi/2}d\theta$

$=\frac{4}{3}\int_0^{2\pi}d\theta=\frac{8}{3}\pi$

42. (a) $I_z=\int_0^{2\pi}\int_0^1\int_0^{\sqrt{1-r^2}}r^2\,dz\,r\,dr\,d\theta$

(b) $I_z=\int_0^{2\pi}\int_0^{\pi/2}\int_0^1(\rho^2\sin^2\phi)(\rho^2\sin\phi)\,d\rho\,d\phi\,d\theta$

Since $r^2=x^2+y^2=\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta=\rho^2\sin^2\phi$

(c) $I_z=\int_0^{2\pi}\int_0^{\pi/2}\frac{1}{5}\sin^3\phi\,d\phi\,d\theta=\frac{1}{5}\int_0^{2\pi}\left(\left[-\frac{\sin^2\phi\cos\phi}{3}\right]_0^{\pi/2}+\frac{2}{3}\int_0^{\pi/2}\sin\phi\,d\phi\right)d\theta$

$=\frac{2}{15}\int_0^{2\pi}\left[-\cos\phi\right]_0^{\pi/2}d\theta=\frac{2}{15}(2\pi)=\frac{4\pi}{15}$

62. let $z = x^2+y^2 = 2-z^2 \Rightarrow z=1$ or $z=-2 \Rightarrow$ the sphere and paraboloid intersect at $\{(x,y,z) \mid z=1,\ x^2+y^2=1\}$ since $z \geq 0$.

$\Rightarrow$ the volume $V = 4\int_0^{\pi/2}\int_0^1\int_{r^2}^{\sqrt{2-r^2}} dz\, r\,dr\,d\theta = 4\int_0^{\pi/2}\int_0^1 [r(2-r^2)^{1/2} - r^3]\,dr\,d\theta$

$= 4\int_0^{\pi/2}\left[-\frac{1}{3}(2-r^2)^{3/2} - \frac{r^4}{4}\right]_0^1 d\theta = 4\int_0^{\pi/2}\left(\frac{2\sqrt{2}}{3} - \frac{7}{12}\right)d\theta = \frac{\pi(8\sqrt{2}-7)}{6}$

66. average $= \dfrac{\int_0^{2\pi}\int_0^{\pi/2}\int_0^1 (\rho\cos\phi)(\rho^2\sin\phi)\,d\rho\,d\phi\,d\theta}{\int_0^{2\pi}\int_0^{\pi/2}\int_0^1 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta} = \dfrac{1}{\left(\frac{2}{3}\pi\right)}\int_0^{2\pi}\int_0^{\pi/2}\int_0^1 \rho^3\cos\phi\sin\phi\,d\rho\,d\phi\,d\theta$

$= \frac{3}{8\pi}\int_0^{2\pi}\int_0^{\pi/2}\cos\phi\sin\phi\,d\phi\,d\theta = \frac{3}{8\pi}\int_0^{2\pi}\left[\frac{\sin^2\phi}{2}\right]_0^{\pi/2} d\theta$

$= \frac{3}{16\pi}\int_0^{2\pi} d\theta = \frac{3}{8}$